Maximum Likelihood Estimation of Compound-Gaussian Clutter and Target Parameters

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Abstract The compound-Gaussian model is often used in radar signal processing to describe the heavy-tailed clutter distribution. The important problems in compound-Gaussian clutter modeling are choosing the texture distribution and estimating its parameters. Many texture distribution models have been proposed [1], [2] and their parameters were typically estimated using the (statistically suboptimal) method of moments, see [2]. In this paper, we develop maximum likelihood (ML) methods for jointly estimating target and clutter parameters in compound-Gaussian clutter. In particular, we estimate (i) the complex target amplitudes, (ii) covariance matrix of the speckle component, and (iii) the texture-distribution parameters. Several existing texture models are considered: (i) gamma (leading to the well-known K clutter distribution [1], [2]), (ii) lognormal, and (iii) Weibull. Motivated by the robust regression model in [3], we also develop a complex multivariate t distribution model for the clutter. We utilize the expectation-maximization (EM) algorithm to estimate the unknown parameters. Numerical integration is typically needed to compute the conditional expectations in the expectation (E) step of the EM algorithm; here, we employ the Gauss quadratures to perform this integration. Interestingly, the proposed complex multivariate t distribution model does not require numerical integration, allowing for remarkably simple estimation and detection algorithms. We will also compute Cramer-Rao bounds (CRBs) for the unknown parameters and demonstrate the performances of the proposed methods via numerical simulations.

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MAXIMUM LIKELIHOOD ESTIMATION OF COMPOUND-GAUSSIAN CLUTTER AND TARGET PARAMETERS

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ABSTRACT

Compound-Gaussian models are used in radar signal processing to describe heavy-tailed clutter distributions. Important problems in compound-Gaussian clutter modeling are: choosing the texture distribution and estimating its parameters. Many texture distributions have been studied and their parameters were typically estimated using statistically suboptimal approaches. We develop maximum likelihood (ML) methods for jointly estimating target and clutter parameters in compound-Gaussian clutter using radar array measurements. In particular, we estimate* (i) *complex target amplitudes,* (ii) *spatial covariance matrix of the speckle component, and* (iii) *texture distribution parameters. We consider two existing texture models, lognormal and gamma, and propose an inverse-gamma texture model, leading to a complex multivariate t clutter distribution. Parameter-expanded expectation-maximization (PX-EM) algorithms are developed to compute the ML estimates of the unknown parameters. For lognormal and gamma textures, Gauss quadratures are utilized to implement the estimation algorithms, whereas the inverse-gamma texture model does not require numerical integration, thus yielding remarkably simple estimators. We study the performance of the proposed methods via numerical simulations.

1. INTRODUCTION

Compound-Gaussian models have been used to characterize heavy-tailed clutter distributions in radar as well as to model speech waveforms, fast fading channels, and various radio propagation channel disturbances, see [1] and references therein. Important problems in compound-Gaussian clutter modeling are: choosing the texture distribution and estimating its parameters. Many texture distributions have been studied and their parameters were typically estimated using (statistically suboptimal) method of moments, see [2]. In this paper, we present maximum likelihood (ML) methods for estimating target and clutter parameters in compound-Gaussian clutter.

In Section 2, we introduce the measurement scenarios with lognormal [2], gamma [2]–[4], and inverse-gamma texture models[1]. For these three models, we develop parameter-expanded expectation-maximization (PX-EM) algorithms to compute the ML estimates of the unknown parameters (see Sections 3.1, 3.2, and 3.3, respectively) and evaluate their performance in Section 4.

2. MEASUREMENT MODEL

We extend the radar array measurement model in [6] to account for compound-Gaussian clutter. Assume that an n-element radar array receives P pulse returns, where each pulse provides N range-gate samples. We collect the spatio-temporal data from the tth range gate into a vector $\boldsymbol{y}(t)$ of size $m = nP$ and model $\boldsymbol{y}(t)$ as[2] (see [6] and [7])

$$\boldsymbol{y}(t) = AX\boldsymbol{\phi}(t) + \boldsymbol{e}(t), \quad t = 1, \ldots, N, \tag{1}$$

where A is an $m \times r$ spatio-temporal steering matrix of the targets, $\Phi = [\boldsymbol{\phi}(1), \boldsymbol{\phi}(2), \ldots, \boldsymbol{\phi}(N)]$ is the temporal response matrix, X is an $r \times d$ matrix of unknown complex amplitudes of the targets, and $\boldsymbol{e}(t)$ is additive noise. Here, we assume that the additive noise vectors $\boldsymbol{e}(t)$ are independent, identically distributed (i.i.d.) and come from a *compound-Gaussian* probability distribution, see e.g. [1]–[4] and [8]–[10]. We now represent the above measurement scenario using the following hierarchical model: $\boldsymbol{y}(t)$ are conditionally independent random vectors with probability density functions (pdfs):

$$\begin{aligned} p_{\boldsymbol{y}|u}(\boldsymbol{y}(t)\,|\,u(t); X, \Sigma) \quad &= \quad \exp\Big\{ - [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]^H \\ &\cdot [u(t)\Sigma]^{-1} \cdot [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)] \Big\} \Big/ |\pi u(t) \Sigma|, \end{aligned} \tag{2}$$

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[1] The proposed inverse-gamma texture model leads to a complex multivariate t clutter distribution, which generalizes the multivariate t distribution in e.g. [5] to the complex-data measurement scenario.

[2] A special case of the model (1) for rank-one targets (i.e. scalar X) in compound-Gaussian clutter was considered in [8].

where "^{H}" denotes the Hermitian (conjugate) transpose, Σ is the (unknown) covariance matrix of the *speckle component*, and $u(t)$, $t = 1, 2, \ldots, N$ are the unobserved *texture components* (powers), modeled as i.i.d. non-negative random variables. We consider the following texture distributions:

- **lognormal:** $\ln u(t)$ follow a Gaussian distribution,
- **gamma:** $u(t)$ follow a gamma distribution, and
- **inverse gamma:** $1/u(t)$ follow a gamma distribution.

Our goal is to compute the ML estimates of the complex amplitude matrix X, speckle covariance matrix Σ, and texture distribution parameters from the measurements $\boldsymbol{y} = [\boldsymbol{y}(1)^T, \boldsymbol{y}(2)^T, \ldots, \boldsymbol{y}(N)^T]^T$. In the following, we present parameter-expanded expectation-maximization (PX-EM) algorithms for ML estimation of these parameters under the above three texture models. The PX-EM algorithms share the same monotonic convergence properties as the "classical" expectation-maximization (EM) algorithms, see [11, Theorem 1]. They outperform the EM algorithms in the global rate of convergence, see [11, Theorem 2].

3. ML ESTIMATION

3.1. PX-EM Algorithm for Lognormal Texture

Assume that the unobserved texture component follows a lognormal distribution (see also [2]); equivalently, $\beta(t) = \ln u(t)$, $t = 1, 2, \ldots, N$ are Gaussian. We further assume that $\beta(t)$ have zero mean and unknown variance σ_β^2; hence, the unknown parameters are $\boldsymbol{\theta} = \{X, \Sigma, \sigma_\beta^2\}$. [Note that the mean of $\beta(t)$ can be chosen arbitrarily because the speckle covariance matrix Σ is *unknown*.]

We now develop a PX-EM algorithm to estimate $\boldsymbol{\theta}$ by treating $\beta(t)$, $t = 1, 2, \ldots, N$ as the *unobserved* (or missing) data and adding an auxiliary (dummy) parameter μ_β [the mean of $\beta(t)$] to the set of parameters $\boldsymbol{\theta}$. Under the *expanded* parameter model, the pdf of $\beta(t)$ is

$$p_\beta(\beta(t); \mu_\beta, \sigma_\beta^2) = \frac{1}{\sigma_\beta\sqrt{2\pi}} \exp\{-[\beta(t) - \mu_\beta]^2/(2\sigma_\beta^2)\}. \quad (3)$$

We also define the augmented (expanded) set of parameters $\boldsymbol{\theta}_\mathrm{a} = \{X, \Sigma_\mathrm{a}, \sigma_\beta^2, \mu_\beta\}$, where $\Sigma = \exp(\mu_\beta) \cdot \Sigma_\mathrm{a}$. The PX-EM algorithm for this model consists of iterating between the following PX-E and PX-M steps:

PX-E Step: Compute

$$\begin{aligned} T_1(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \Big\{ \boldsymbol{y}(t) \boldsymbol{\phi}(t)^H \\ &\quad \cdot \mathrm{E}_{\beta|\boldsymbol{y}}[\exp[-\beta(t)] \,|\, \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}^{(i)}] \Big\}, \end{aligned} \quad (4a)$$

$$\begin{aligned} T_2(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \Big\{ \boldsymbol{y}(t) \boldsymbol{y}(t)^H \\ &\quad \cdot \mathrm{E}_{\beta|\boldsymbol{y}}[\exp[-\beta(t)] \,|\, \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}^{(i)}] \Big\}, \end{aligned} \quad (4b)$$

$$\begin{aligned} T_3(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \Big\{ \boldsymbol{\phi}(t) \boldsymbol{\phi}(t)^H \\ &\quad \cdot \mathrm{E}_{\beta|\boldsymbol{y}}[\exp[-\beta(t)] \,|\, \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}^{(i)}] \Big\}, \end{aligned} \quad (4c)$$

$$t_4(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}) = \frac{1}{N} \cdot \sum_{t=1}^{N} \mathrm{E}_{\beta|\boldsymbol{y}}[\beta^2(t) \,|\, \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}^{(i)}], \quad (4d)$$

$$t_5(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}) = \frac{1}{N} \cdot \sum_{t=1}^{N} \mathrm{E}_{\beta|\boldsymbol{y}}[\beta(t) \,|\, \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}^{(i)}], \quad (4e)$$

where

$$\boldsymbol{\theta}_\mathrm{a}^{(i)} = \{X^{(i)}, \Sigma_\mathrm{a}^{(i)}, (\sigma_\beta^2)^{(i)}, \mu_\beta^{(i)}\} \quad (4f)$$

is the estimate of $\boldsymbol{\theta}_\mathrm{a}$ in the ith iteration and (4a)–(4e) are computed using (5) (below).

PX-M Step: Compute

$$\begin{aligned} X^{(i+1)} &= [A^H (S^{(i)})^{-1} A]^{-1} A^H (S^{(i)})^{-1} \\ &\quad \cdot T_1(\boldsymbol{y}, \boldsymbol{\theta}_\mathrm{a}^{(i)})\, T_3(\boldsymbol{y}, \boldsymbol{\theta}_\mathrm{a}^{(i)})^{-1}, \end{aligned} \quad (6a)$$

$$\begin{aligned} \Sigma_\mathrm{a}^{(i+1)} &= S^{(i)} + [I_m - Q^{(i)} (S^{(i)})^{-1}]\, T_1(\boldsymbol{y}, \boldsymbol{\theta}_\mathrm{a}^{(i)}) \\ &\quad \cdot T_3(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)})^{-1} T_1(\boldsymbol{y}, \boldsymbol{\theta}_\mathrm{a}^{(i)})^H\, [I_m - Q^{(i)} (S^{(i)})^{-1}]^H, \end{aligned} \quad (6b)$$

$$\mu_\beta^{(i+1)} = t_5(\boldsymbol{y}; \boldsymbol{\theta}_\mathrm{a}^{(i)}), \quad (6c)$$

$$(\sigma_\beta^2)^{(i+1)} = t_4(\boldsymbol{y}, \boldsymbol{\theta}_\mathrm{a}^{(i)}) - (\mu_\beta^{(i+1)})^2, \quad (6d)$$

$$\Sigma^{(i+1)} = \exp(\mu_\beta^{(i+1)}) \cdot \Sigma_\mathrm{a}^{(i+1)}, \quad (6e)$$

$$\begin{aligned} &\mathrm{E}_{\beta|\boldsymbol{y}}[\exp[-\beta(t)] | \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}] \\ &\approx \frac{\sum_{l=1}^{L} h_l \cdot \upsilon_l(\mu_\beta, \sigma_\beta^2)^{-(m+1)} \cdot \exp\{-\upsilon_l(\mu_\beta, \sigma_\beta^2)^{-1} \cdot [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]^H \Sigma_\mathrm{a}^{-1} [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]\}}{\sum_{l=1}^{L} h_l \cdot \upsilon_l(\mu_\beta, \sigma_\beta^2)^{-m} \cdot \exp\{-\upsilon_l(\mu_\beta, \sigma_\beta^2)^{-1} \cdot [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]^H \Sigma_\mathrm{a}^{-1} [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]\}}, \end{aligned} \quad (5a)$$

$$\begin{aligned} &\mathrm{E}_{\beta|\boldsymbol{y}}[\beta^k(t) | \boldsymbol{y}(t); \boldsymbol{\theta}_\mathrm{a}] \\ &\approx \frac{\sum_{l=1}^{L} h_l \cdot (\sqrt{2}\sigma_\beta x_l + \mu_\beta)^k \cdot \upsilon_l(\mu_\beta, \sigma_\beta^2)^{-m} \cdot \exp\{-\upsilon_l(\mu_\beta, \sigma_\beta^2)^{-1} \cdot [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]^H \Sigma_\mathrm{a}^{-1} [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]\}}{\sum_{l=1}^{L} h_l \cdot \upsilon_l(\mu_\beta, \sigma_\beta^2)^{-m} \cdot \exp\{-\upsilon_l(\mu_\beta, \sigma_\beta^2)^{-1} \cdot [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]^H \Sigma_\mathrm{a}^{-1} [\boldsymbol{y}(t) - AX\boldsymbol{\phi}(t)]\}}, \end{aligned} \quad (5b)$$

where $\upsilon_l(\mu_\beta, \sigma_\beta^2) = \exp(\sqrt{2}\sigma_\beta x_l + \mu_\beta)$ and $k \in \{1, 2\}$.

where

$$
\begin{aligned}
S^{(i)} &= T_2(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) \\
&\quad - T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) \cdot T_3(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^{-1} \cdot T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^H, \qquad (6f) \\
Q^{(i)} &= A[A^H (S^{(i)})^{-1} A]^{-1} A^H. \qquad (6g)
\end{aligned}
$$

The above iteration is performed until $X^{(i)}$, $\Sigma^{(i)}$, and $(\sigma_\beta^2)^{(i)}$ converge. Here, I_m denotes the identity matrix of size m,

We now discuss computing the conditional expectations in (5). First, recall the Gauss-Hermite quadrature formula [12, Ch. 5.3]:

$$
\int_{-\infty}^{\infty} f(x) \cdot \exp(-x^2)\, dx \approx \sum_{l=1}^{L} h_l\, f(x_l), \qquad (7)
$$

where $f(x)$ is an arbitrary real function, L is the quadrature order (determining approximation accuracy), and x_l and $h_l,\ l = 1, 2, \ldots, L$ are the abscissas and weights of the Gauss-Hermite quadrature (respectively), tabulated in e.g. [13]. Using (7), the Bayes rule, equations (2) and (3), and change-of-variable transformation $x = (\beta - \mu_\beta)/(\sqrt{2}\sigma_\beta)$, we obtain the approximate expression in (5).

3.2. PX-EM Algorithm for Gamma Texture

We now model the texture components $u(t),\ t = 1, 2, \ldots, N$ as gamma random variables with mean one (as in e.g. [4]) and unknown shape parameter $\nu > 0$; hence, the unknown parameters are $\boldsymbol{\theta} = \{X, \Sigma, \nu\}$. (The shape parameter ν is also known as the Nakagami-m parameter in the communications literature, see e.g. [14, Ch. 2.2.1.4].) This choice of the texture distribution leads to the well-known K clutter model, see [2] and [4] and references therein.

We develop a PX-EM algorithm to estimate $\boldsymbol{\theta}$ by treating $u(t),\ t = 1, 2, \ldots, N$ as the unobserved data and adding an auxiliary parameter μ_u [the mean of $u(t)$] to the set of parameters $\boldsymbol{\theta}$. Under this expanded model, the pdf of $u(t)$ is [for $u(t) \geq 0$]

$$
p_u(u(t); \nu, \mu) = \frac{1}{\Gamma(\nu)} \Big(\frac{\nu}{\mu_u}\Big)^{\nu} u(t)^{\nu - 1} \exp\big[- \nu u(t)/\mu_u \big] \qquad (8)
$$

where $\Gamma(\cdot)$ denotes the gamma function. Hence, the augmented parameter set is $\boldsymbol{\theta}_{\mathrm{a}} = \{X, \Sigma_{\mathrm{a}}, \nu, \mu_u\}$, where Σ_{a} and Σ are related as follows: $\Sigma = \mu_u \cdot \Sigma_{\mathrm{a}}$. The PX-EM algorithm for the above expanded model consists of iterating between the following PX-E and PX-M steps:

PX-E Step: Compute

$$
\begin{aligned}
T_1(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \boldsymbol{y}(t) \boldsymbol{\phi}(t)^H \\
&\qquad \cdot \mathrm{E}_{\,u|\boldsymbol{y}}[\, u(t)^{-1} \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}^{(i)}], \qquad (9a) \\
T_2(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \boldsymbol{y}(t) \boldsymbol{y}(t)^H \\
&\qquad \cdot \mathrm{E}_{\,u|\boldsymbol{y}}[\, u(t)^{-1} \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}^{(i)}], \qquad (9b) \\
T_3(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \boldsymbol{\phi}(t) \boldsymbol{\phi}(t)^H \\
&\qquad \cdot \mathrm{E}_{\,u|\boldsymbol{y}}[\, u(t)^{-1} \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}^{(i)}], \qquad (9c) \\
t_4(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \mathrm{E}_{\,u|\boldsymbol{y}}[\, \ln u(t) \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}^{(i)}], \qquad (9d) \\
t_5(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) &= \frac{1}{N} \cdot \sum_{t=1}^{N} \mathrm{E}_{\,u|\boldsymbol{y}}[\, u(t) \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}^{(i)}], \qquad (9e)
\end{aligned}
$$

where $\boldsymbol{\theta}_{\mathrm{a}}^{(i)} = \{X^{(i)}, \Sigma_{\mathrm{a}}^{(i)}, \nu^{(i)}, \mu_u^{(i)}\}$ is the estimate of $\boldsymbol{\theta}_{\mathrm{a}}$ in the ith iteration and (9a)–(9e) are computed using (10) (below) with $g(u(t)) = u(t)^{-1}, \ln u(t)$, and $u(t)$.

PX-M Step: Compute

$$
\begin{aligned}
X^{(i+1)} &= [A^H (S^{(i)})^{-1} A]^{-1} A^H (S^{(i)})^{-1} \\
&\qquad \cdot T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)})\, T_3(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^{-1}, \qquad (11a) \\
\Sigma_{\mathrm{a}}^{(i+1)} &= S^{(i)} + [I_m - Q^{(i)} (S^{(i)})^{-1}]\, T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) \\
&\quad \cdot T_3(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^{-1} T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^H\, [I_m - Q^{(i)} (S^{(i)})^{-1}]^H, \qquad (11b) \\
\mu_u^{(i+1)} &= t_5(\boldsymbol{y}, \boldsymbol{\theta}^{(i)}), \qquad (11c) \\
\Sigma^{(i+1)} &= \mu_u^{(i+1)} \cdot \Sigma_{\mathrm{a}}^{(i+1)}, \qquad (11d)
\end{aligned}
$$

where

$$
\begin{aligned}
S^{(i)} &= T_2(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) \\
&\quad - T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)}) \cdot T_3(\boldsymbol{y}; \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^{-1} \cdot T_1(\boldsymbol{y}, \boldsymbol{\theta}_{\mathrm{a}}^{(i)})^H, \qquad (11e) \\
Q^{(i)} &= A\, [A^H (S^{(i)})^{-1} A]^{-1} A^H, \qquad (11f)
\end{aligned}
$$

and find $\nu^{(i+1)}$ that maximizes

$$
\begin{aligned}
\nu^{(i+1)} = \arg\max_{\nu} \Big\{ &- \ln \Gamma(\nu) + \nu \ln \nu - \nu \ln[t_5(\boldsymbol{y}, \boldsymbol{\theta}^{(i)})] \\
&+ \nu t_4(\boldsymbol{y}, \boldsymbol{\theta}^{(i)}) - \nu \Big\}. \qquad (11c)
\end{aligned}
$$

The above iteration is performed until $X^{(i)}$, $\Sigma^{(i)}$, and $\nu^{(i)}$ converge. The conditional-expectation expression (10) is obtained by using the Bayes rule, equations (2) and (8), and change-of-variable transformation $x = \nu u/\mu$. The integrals

$$
\mathrm{E}_{\,u|\boldsymbol{y}}\big[g(u(t)) \mid \boldsymbol{y}(t); \boldsymbol{\theta}_{\mathrm{a}}\big] = \frac{\int_0^{\infty} g(x\mu_u/\nu) \cdot p_{\boldsymbol{y}|u}(y(t) \mid x\mu_u/\nu\,;\, X, \Sigma_{\mathrm{a}}) \cdot x^{\nu-1}\, \exp(-x)\; dx}{\int_0^{\infty} p_{\boldsymbol{y}|u}(y(t) \mid x\mu_u/\nu\,;\, X, \Sigma_{\mathrm{a}}) \cdot x^{\nu-1}\, \exp(-x)\; dx}. \qquad (10)
$$

in the numerator and denominator of (10) are efficiently and accurately evaluated using the generalized Gauss-Laguerre quadrature formula (see [12, Ch. 5.3]):

$$\int_0^\infty f(x)\cdot x^{\nu-1}\exp(-x)\,dx \approx \sum_{l=1}^{L} w_l(\nu-1)\, f(x_l(\nu-1)), \tag{12}$$

where $f(x)$ is an arbitrary real function, L is the quadrature order, and $x_l(\nu-1)$ and $w_l(\nu-1),\ l=1,2,\dots,L$ are the abscissas and weights of the generalized Gauss-Laguerre quadrature with parameter $\nu-1$.

The computation of $\nu^{(i+1)}$ requires maximizing (11c), which is performed using the Newton-Raphson method (embedded within the "outer" EM iteration, similar to [15]).

3.3. PX-EM Algorithm for Inverse Gamma Texture

We now propose a *complex multivariate t distribution model* for the clutter and apply it to the measurement scenario in Section 2. A similar clutter model was briefly discussed in [10, Sec. IV.B.3], where it was also referred to as the *generalized Cauchy distribution*. Assume that $w(t)=u(t)^{-1}$, $t=1,2,\dots,N$ are gamma random variables with mean one and unknown shape parameter $\nu>0$. Consequently, $u(t)$ follows an *inverse gamma* distribution and the conditional distribution of $\boldsymbol{y}(t)$ given $w(t)$ is $p_{\boldsymbol{y}|u}(\boldsymbol{y}(t)|w(t)^{-1};X,\Sigma)$, see also (2). Integrating the unobserved data $w(t)$ out, we obtain a *closed-form* expression for the marginal pdf of $\boldsymbol{y}(t)$:

$$\begin{aligned}p_{\boldsymbol{y}}(\boldsymbol{y}(t);X,\Sigma,\nu) &= \frac{\Gamma\big(\nu+m\big)}{|\pi\Sigma|\cdot\Gamma\big(\nu\big)\cdot\nu^m}\cdot\Big\{1+\\ &\cdot[\boldsymbol{y}(t)-AX\boldsymbol{\phi}(t)]^H\,\Sigma^{-1}\,[\boldsymbol{y}(t)-AX\boldsymbol{\phi}(t)]/\nu\Big\}^{-\nu-m},\end{aligned} \tag{13}$$

which is the *complex multivariate t distribution* with location vector $AX\boldsymbol{\phi}(t)$, scale matrix Σ, and shape parameter ν. Here, the unknown parameters are $\boldsymbol{\theta}=\{X,\Sigma,\nu\}$. We first estimate X and Σ assuming that the shape parameter ν is *known* and then discuss the estimation of ν.

Known ν: For a fixed ν, we derive a PX-EM algorithm for estimating X and Σ by treating $w(t)$, $t=1,2,\dots,N$ as the unobserved data and adding an auxiliary mean parameter for $w(t)$, similar to the lognormal and gamma cases discussed in Sections 3.1 and 3.2. Here, the resulting PX-EM algorithm consists of iterating between the following PX-E and PX-M steps:

PX-E Step: Compute

$$\begin{aligned}\widehat{w}^{(i)}(t) &= (\nu+m)\cdot\Big\{\nu+[\boldsymbol{y}(t)-AX^{(i)}\boldsymbol{\phi}(t)]^H\\ &\cdot[\Sigma^{(i)}]^{-1}\,[\boldsymbol{y}(t)-AX^{(i)}\boldsymbol{\phi}(t)]\Big\}^{-1}\end{aligned} \tag{14a}$$

for $t=1,2,\dots,N$ and

$$T_1^{(i)} = \frac{1}{N}\cdot\sum_{t=1}^{N}\boldsymbol{y}(t)\boldsymbol{\phi}(t)^H\cdot\widehat{w}^{(i)}(t), \tag{14b}$$

$$T_2^{(i)} = \frac{1}{N}\cdot\sum_{t=1}^{N}\boldsymbol{y}(t)\boldsymbol{y}(t)^H\cdot\widehat{w}^{(i)}(t), \tag{14c}$$

$$T_3^{(i)} = \frac{1}{N}\cdot\sum_{t=1}^{N}\boldsymbol{\phi}(t)\boldsymbol{\phi}(t)^H\cdot\widehat{w}^{(i)}(t). \tag{14d}$$

PX-M Step: Compute

$$X^{(i+1)} = [A^H(S^{(i)})^{-1}A]^{-1}A^H(S^{(i)})^{-1}\,T_1^{(i)}(T_3^{(i)})^{-1}, \tag{15a}$$

$$\begin{aligned}\Sigma^{(i+1)} &= \Big\{S^{(i)}+[I_m-Q^{(i)}(S^{(i)})^{-1}]\cdot T_1^{(i)}\,(T_3^{(i)})^{-1}\\ &\cdot(T_1^{(i)})^H\,[I_m-Q^{(i)}(S^{(i)})^{-1}]^H\Big\}\Big/\Big[\frac{1}{N}\sum_{t=1}^{N}\widehat{w}^{(i)}(t)\Big],\end{aligned} \tag{15b}$$

where

$$S^{(i)} = T_2^{(i)}-T_1^{(i)}\,(T_3^{(i)})^{-1}\,(T_1^{(i)})^H, \tag{15c}$$

$$Q^{(i)} = A\,[A^H(S^{(i)})^{-1}A]^{-1}\,A^H. \tag{15d}$$

The above iteration is performed until $X^{(i)}$ and $\Sigma^{(i)}$ converge. Denote by $X^{(\infty)}(\nu)$ and $\Sigma^{(\infty)}(\nu)$ the estimates of X and Σ obtained upon convergence, where we emphasize their dependence on ν.

Unknown ν: We compute the ML estimate of ν by maximizing the observed-data log-likelihood function concentrated with respect to $\widehat{X}(\nu)$ and $\widehat{\Sigma}(\nu)$:

$$\widehat{\nu} = \arg\max_{\nu}\sum_{t=1}^{N}\ln p_{\boldsymbol{y}}(\boldsymbol{y}(t);X^{(\infty)}(\nu),\Sigma^{(\infty)}(\nu),\nu), \tag{16}$$

see also (13).

4. SIMULATION RESULTS

The numerical example presented here assess the estimation accuracy of the ML estimates of X,Σ, and the shape parameters of the texture components. We consider a measurement scenario with a 3-element radar array and $P=3$ pulses, implying that $m=9$. We selected a rank-one target scenario with $\boldsymbol{\phi}(t)=1$, $t=1,2,\dots,N$, complex target amplitude $X=0.207\cdot\exp(j\pi/7)$, and

$$A=\boldsymbol{b}(\varpi)\otimes\boldsymbol{a}(\vartheta),$$

where $\boldsymbol{b}(\varpi)=[1,\exp(j2\pi\varpi),\exp(j4\pi\varpi)]^T$ with *normalized Doppler frequency* $\varpi=0.42$, and $\boldsymbol{a}(\vartheta)=[1,\exp(j2\pi\vartheta),\exp(j4\pi\vartheta)]^T$ with *spatial frequency* $\vartheta=0.926$. Here, $\otimes$ and "^{T}" denote the Kronecker product and transpose, respectively. The speckle covariance matrix Σ was generated

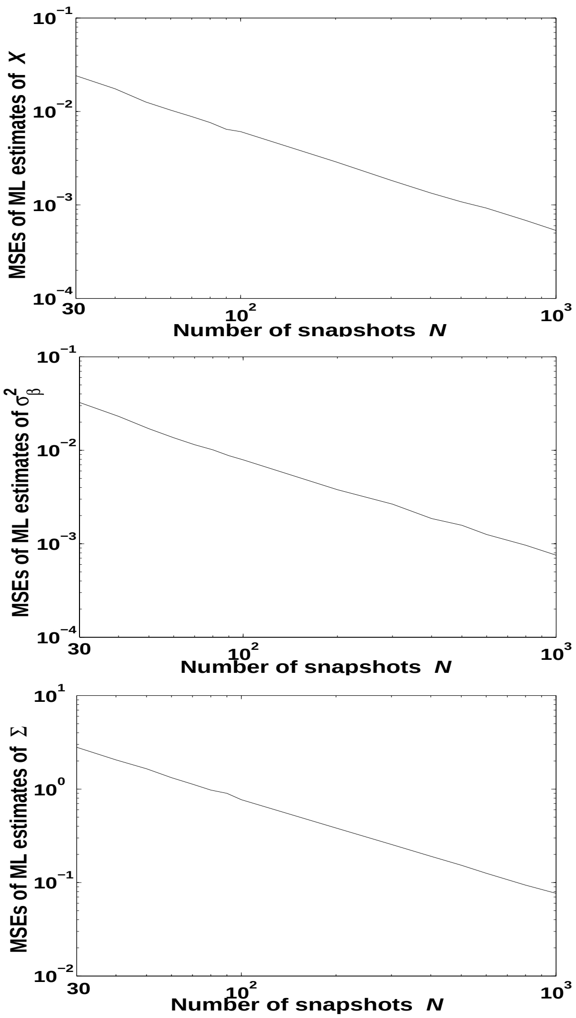

Figure 1: MSEs for the ML estimates of X, σ_β^2, and Σ as functions of N under the lognormal texture model.

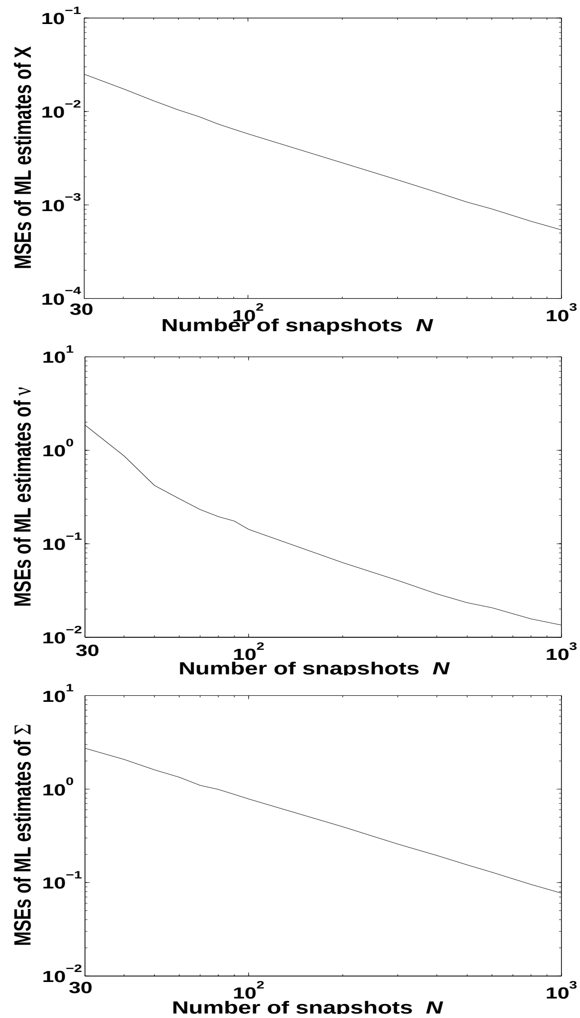

Figure 2: MSEs for the ML estimates of X, ν, and Σ as functions of N under the gamma texture model.

using a model similar to that in [16, Sec. 2.6] with 1000 patches; the diagonal elements of Σ were 10.17. Our performance metric is the mean-square error (MSE) of an estimator, calculated using 2500 independent trials. The order of the Gauss-Hermite and generalized Gauss-Laguerre quadratures was $L = 20$.

We first study the performance of the ML estimation of the clutter and target parameters for lognormal texture in Section 3.1. Here, the variance of $\beta(t)$ was set to $\sigma_\beta^2 = 0.5$. Fig. 1 shows the MSEs for the ML estimates of X and σ_β^2 and the average MSE for the ML estimates of the speckle covariance parameters[3], as functions of the number of range samples (snapshots) N.

We now examine the performance of the ML estimation for gamma texture in Section 3.2. We have set the shape parameter to $\nu = 2$. Fig. 2 shows the MSEs for the ML estimates of X and ν and the average MSE for the ML estimates of the speckle covariance parameters, as functions of N.

Finally, we show the performance of the ML estimation for inverse gamma texture in Section 3.2. Here, the shape parameter was set to $\nu = 4$. Fig. 3 shows the MSEs for the ML estimates of X and ν and the average MSE for the ML estimates of the speckle covariance parameters, as functions of N.

[3] In particular, we average the MSEs of the elements of $\mathrm{Re}\{\mathrm{vech}(\Sigma)\}$ and $\mathrm{Im}\{\underline{\mathrm{vech}}(\Sigma)\}^T]^T$. Here, the vech and $\underline{\mathrm{vech}}$ operators create a single column vector by stacking elements below the main diagonal columnwise; vech includes the main diagonal, whereas $\underline{\mathrm{vech}}$ omits it.

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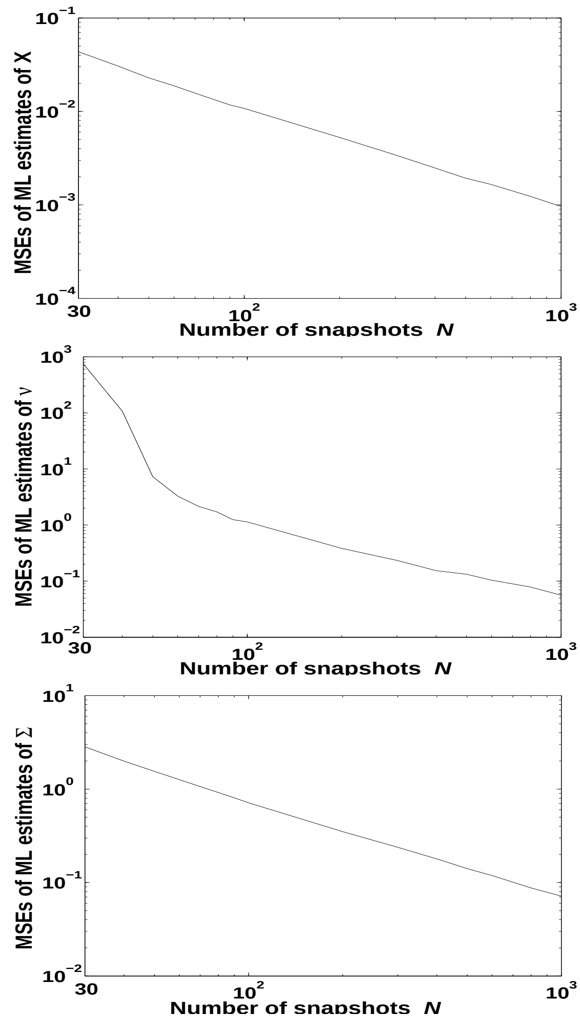

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MAXIMUM LIKELIHOOD ESTIMATION OF COMPOUND-GAUSSIAN CLUTTER AND TARGET PARAMETERS

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Summary

Develop maximum likelihood (ML) methods for jointly estimating target and clutter parameters in compound-Gaussian clutter using radar array measurements.

We estimate:

(i) complex target amplitudes,

(ii) spatial covariance matrix of the speckle component,

(iii) texture distribution parameters.